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ERROR DIAGNOSIS IN INFINITE WALL BOUNDARY CONDITIONS OF SCHRÖDINGER EQUATION USING RANDOM SIGNAL ANALYSIS

CHETAN AMBEKAR[1], JESIL GANDHI[2], HARSH KOTHARI[3] & MURTAZA HAAMID[4]

[1,2,3]Department of Electronics & Telecommunication Engineering, K. J. S. C. O. E, University of Mumbai, Mumbai, India

[4]Department of Computer Engineering, K.J.S.I.E.T, University of Mumbai, Mumbai, India

ABSTRACT

Wireless networking is the branch of communication in which wireless channels are used for the transmission of the signals. There are various terminologies in the field of wireless networking which are taken form quantum physics. Infact quantum physics consists of the signals which are in wireless medium. There are many important theories which are given in the field of quantum mechanics which can be related with the concepts of wireless networks. In this paper we have tried to link the famous Hinesburg Uncertainty Principle and the Schrödinger wave equation with the wireless network in order to find the error in the position, momentum and energy of the network. A new concept is also discussed regarding the error in the probability in which an attempt is made to derive the equation of error with respect to the displacement. In order to derive these equations various concepts of mathematical modeling, Fourier series, Fourier transform, error diagnosis are discussed briefly.

KEYWORDS: Momentum, Wavelets, Duality, Uncertainty

I. INTRODUCTION

Quantum Physics is the branch of science which highly deals with the electrons and other subatomic particle behaving as in the wave nature. Generally, scientists have two views that electron and other subatomic particles has two nature either wave or particle. Several theories were presented by the scientists to prove that electron follows wave nature. Generally in the field of wireless networking, waves are used for the transmission of the signal. The waves which are used here can be compared to the electron and the other sub-atomic particle waves as generated by the wave nature duality.

The paper includes the method through which the concepts of Schrödinger Equation can be modified to be used in wireless medium. The paper also includes the concepts the probability in which the normal distribution of probability is used for the analysis of the theorems. We have tried to come up with a new concept in which the error in probability is introduced. The paper majorly includes the connection of Schrödinger equations with the implementation of the theories of wireless networking. The paper usually deals with the application layer of the wireless modeling. Probability of the energy density is studied between the nodes by considering cases of boundary condition. Error in the probability will be derived by considering the Heisenberg and Schrödinger equation. Normally if we take the case of normal form of distribution we have the step deviation for which a constant probability occurs but generally there is an error in the probability which can be linked through the error in the position, velocity or momentum of the particle.

He is enberg's uncertainty principle, however, is a special case and it refers to wave packets with Gauss distribution. The cases of Schrödinger equations which include the energy quantization, boundary conditions are taken from [1]. The proof of the strong inequality was given by Kennard and Weyl[2]. Later Robertson [3] generalized the correlation for arbitrary observables A and B 1, and Dichburn [4] presented the relation between Heisenberg's fluctuations. Generalized and précised form of Heisenberg's principle was given by Schrödinger [5, 6] and Robertson [7]. The Schrödinger's relation (3) can be expressed in a compact [6] and "very elegant form" through [9]. Ref. [8] explains about the role of noise in increasing the error. Ref [10] gives the information about the classical cases of quantum analogy used in handbooks on experimental physics. Ref. [11] gives the information regarding the matrix analogy for quantum physics. Definition of quantum mechanics is taken from [12].There are two cases of Schrödinger equation as time independent and other as time independent, we take ref. [13] in which the probability distribution of both the cases are studied.

The remainder of the paper is described in the following ways: Section II provides the extension in the principle In the section III, approximation in the error in probability with respect to energy and position is defined and derived by considering the wave equations of Schrödinger wave equation. Finally conclusion is in section IV.

II. EXTENSION OF PRINCIPLES TO WIRELESS MEDIUM

Extending the concepts of Heisenberg Uncertainty principle in the wireless networking we will have the uncertainty in the probability as there is a chance of having an uncertainty in the presence of connection between two nodes. As discussed the connection between two nodes depends on the energy density of the signals present between these signals. There we can say that the change of this probability will be dependent on the value of ΔE as the value of p is related with the value of energy gradient. It has been described regarding the cutoff value which has been required for the connection between the two nodes.

$$\Delta\, p\,.\,\Delta\, E \geq c'$$

Here c' is a constant whose value can be calculated form the practical experiments.

Consider the case of standing wave generation in one dimension in which there are two nodes and they are connected to each other through a string. When an oscillation is provided to this system it has the tendency to generate a standing wave at the ends of the nodes (starting and ending point). This example can be compared with the three dimensional wireless network where if two nodes are connected to each other then they will have wavelets having oscillations along their path due to which travelling waves are generated at the connecting path of both the nodes but at the ends of both the nodes there is a formation of standing waves. There is a property of standing waves that there is no condition of interference among the signals, this acts as a plus point for the wireless networks as due to this there is no cross connection of the signals within the nodes (this is the basic concept which is generally used for the case of no interference within the wavelets).

This equation has a lot of solutions but the value of ψ must be finite, single valued and continuous. By the values of ψ one has the tendency to find out the complete characteristics of the wavelets. The value of ψ^2 represents the probability of the presence of electron at any position over the network, similarly by calculating the value of ψ^2in the wireless network one can get the value of probability of availability of the signal (energy density of the signals).

In this paper we will highly stick to the boundary conditions which are time independent as the study of the distribution of the probability and potential over a fixed boundary. Again two cases arise in this, as whether the potential voltage of the signal is infinite or a limited. Here we will assume that the value of the potential voltage be zero over the fixed boundary and then infinite for the value uptil infinity.

III. ERROR MEASUREMENT

Consider a case in which we have a single particle of mass m confined to within a region $0 < x < L$ with potential energy $V = 0$ bounded by infinitely high potential barriers, i.e. V= ∞ from x $< 0\ and x > L$.the potential experienced by the particle is then:

$$V(x) = 0 \quad 0 < x < L$$

$$= \infty \quad x \geq L, x \leq 0$$

The regions over which the value of voltage is infinite, no wave function will exist over there, due to this the probability distribution over that region doesn't correspond to the connectivity of the nodes also we can say that this range over which the value of V is zero is the available range between the two nodes to be connected. It is clear that the value of the probability present over the (x=0, L) is zero. Therefore, we have the following expressions.

$$\psi(0) = \psi(L) = 0$$

Now by the Schrödinger wave equation we have,

$$\nabla^2\Psi(x) + \frac{8\pi^2 m}{h^2\psi}(E - V)\psi(x) = 0$$

As potential voltage is taken to be zero over the fixed bounded region therefore we have V = 0.Therefore the equation changes to

$$\nabla^2\Psi(x) + \frac{8\pi^2 m}{h^2}E\psi(x) = 0$$

$$\nabla^2\psi(x) = -\frac{8\pi^2 m}{h^2}E\psi(x)$$

To solve this equation consider the coefficient of ψ (x) be m^2. Therefore the equation changes to,

$$\nabla^2\psi(x) + M^2\psi(x) = 0$$

Therefore the value of m will be

$$M = \sqrt{8mE} * \frac{\pi}{h}$$

$$E = \frac{\left(M.\frac{h}{\pi}\right)^2}{8m}$$

We know that the general solution of a two degree equation is;

$$\psi(x) = Asin(Mx) + Bcos(Mx)$$

By implementing the boundary conditions we know that at x = 0,

$$\psi(0) = 0;$$

Therefore if we put the value of x = 0 in the equations then we get the result as B = 0; the solution of the equation is

$\psi(x) = Asin(Mx)$

Again applying the boundary condition for the other extremity as x = L.

$\psi(L) = 0 = Asin(ML);$

Two cases arises as either the value of A is zero or $sin(kx)$ is zero, taking the first case gives the result as there is no particle in the region of sight, taking the second one so we have the varied values of x for which the system will undergo nil value. The values of mx over which the system will undergo zero value.

$ML = n\pi$ Where n = $0, \pm 1, \pm 2, \pm 3 \ldots \ldots$

We take only positive values of n and discard value as zero as this will make system as zero. The negative values of n will produce the same set of instructions as being produced by positive values. Therefore, values of m will be,

$M = n\pi/L$ Where n = $1, 2, 3 \ldots \ldots$

Put the value of M in E.

$$E = \frac{\left(\frac{n\pi}{L} \cdot \frac{h}{\pi}\right)^2}{8m}$$

$$E = \frac{(n.h)^2}{L^2 8m}$$

We apply the normalization method over the differential equation we get the following result. Therefore, the probability distribution over the specified region will be:

$$\psi(x) = Asin\left(\frac{}{L}\right) \; 0 < x < L$$

$$= 0 \; x > L$$

Now if we consider the normal distribution of the probability then the probability function will be as following:

$$\psi(x)_{env} = \sqrt{P_x} = \sqrt{A}_x e^{-\frac{x-x_0}{4\sigma x^2}}$$

On comparing the values of both the functions, we get the following result:

$$A = \sqrt{A_x}$$

$$sin\left(\frac{n\pi x}{L}\right) = e^{-\frac{x-x_0}{4\sigma x^2}}$$

In order to find the error between probability and step deviation, we need to find out the relation of error in x with respect to the error and in step deviation. And then relate this error with the error of function of probability.

Therefore, considering

$$sin\left(\frac{n\pi x}{L}\right) = e^{-\frac{x-x_0}{4\sigma_x{}^2}}$$

Put the value of $x_0 = 0$,

$$sin\left(\frac{n\pi x}{L}\right) = e^x.e^{-\frac{1}{4\sigma_x{}^2}}$$

Take differentiation on both the sides to find the error in x with respect to σ_x.

$$\frac{\Delta \mathrm{x} \cos\left(\frac{n\pi x}{L}\right)}{\sin\left(\frac{n\pi x}{L}\right)} = \frac{3x_0}{4\sigma_x{}^3}\Delta\sigma_x - \frac{3x}{4\sigma_x{}^3}\Delta\sigma_x - \frac{\Delta x}{4\sigma_x{}^2}$$

$$\left(\cot\left(\frac{n\pi x}{L}\right) + \frac{1}{4\sigma_x{}^2}\right)\Delta \mathrm{x} = \left(\frac{3(x_0 - x)}{4\sigma_x{}^3}\right)\Delta\sigma_x$$

$$\Delta \mathrm{x} = \frac{\left(\frac{3(x_0-x)}{4\sigma_x{}^3}\right)}{\left(\cot\left(\frac{n\pi x}{L}\right) + \frac{1}{4\sigma_x{}^2}\right)}\Delta\sigma_x$$

This relation describes the relation between the step deviation error and error in distance. The relation between the error in position and error in probability is also described below.

We have

$$P_x = A\left(sin^2\left(\frac{n\pi x}{L}\right)\right)$$

Take log on both sides of the equation.

$$\log P_x = \log A + 2\log\sin\left(\frac{n\pi x}{L}\right)$$

Take differentiation on both the sides to find the error between probability and x.

$$\Delta P_x = 2.\frac{\cos\left(\frac{n\pi x}{L}\right)}{\sin\left(\frac{n\pi x}{L}\right)}.\Delta x = 2.\cot\left(\frac{n\pi x}{L}\right).\Delta x$$

Now take the value of Δx from the previous equation and put in this equation to get the error in probability in terms of error in step deviation.

$$\mathrm{x} = \frac{\left(\frac{3(x_0-x)}{4\sigma_x{}^3}\right)}{\left(\cot\left(\frac{n\pi x}{L}\right) + \frac{1}{4\sigma_x{}^2}\right)}\Delta\sigma_x$$

On substituting the values of $\Delta\sigma_x$ we get the error in probability.

$$\Delta P_x = 2.\cot\left(\frac{n\pi x}{L}\right)\frac{\left(\frac{3(x_0-x)}{4\sigma_x{}^3}\right)}{\left(\cot\left(\frac{n\pi x}{L}\right) + \frac{1}{4\sigma_x{}^2}\right)}\Delta\sigma_x$$

$$\Delta P_x = \frac{6(x_0 - x)}{\sigma_x\left(4\sigma_x{}^2 + \tan\left(\frac{n\pi x}{L}\right)\right)}\Delta\sigma_x$$

Thus we have defined a relation between ΔP and $\Delta\sigma_x$ that is the change in the value of step deviation with the error in probability, this equation could be used in order to find the relation between change in probability with the change in energy and change in momentum of the particle. Let, a variable as C

$$C = \frac{6(x_0 - x)}{\sigma_x\left(4\sigma_x{}^2 + \tan\left(\frac{n\pi x}{L}\right)\right)}$$

$$\Delta P_x = C.\Delta\sigma_x$$

Now, we can relate the error in p by others as by the given equations.

$$\Delta\, P\,.\Delta\, p \geq C/4\pi$$

$$\Delta\, \mathrm{P}.\Delta\, v \geq C/4\pi m$$

Therefore from the above equations, the error in p is calculated in terms of Δx. In the equation, there is an increase in the value of σ_x and then the error in p is reduced. Consider the case if $\sigma_x >> 0$, then approximately we take the value of $\sigma_x = \infty$. If we put the value of σ_x as infinite in the above equation then following changes occurs.

$$\Delta P = \Delta\sigma_x.\frac{6(x_0 - x)}{\infty\left(4\infty^2 + \tan\left(\frac{n\pi x}{L}\right)\right)}$$

$$\Delta P \cong 0$$

From the above equation it is clear that the error reduces if there is an increase in the value of σ_x. The error is reduced as if there is a decreasing exponential curve and the expression contains σ_x in the denominator. Therefore we can assume for higher value of step deviation the value of the error is very less.

IV. CONCLUSIONS

The paper consist a brief introduction about the error in the probability conditions in the infinite wall boundary conditions of Schrödinger equation. From the given equations, it is clear that the error in the probability is inversely proportional to step deviation. Due to which if the value of step deviation is high the error in probability is low.

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